NAME _____ DATE _____

4–5 Factoring Polynomials

Objective: To factor polynomials by using the GCF, by recognizing special products, and by grouping terms.

Vocabulary

Factor a polynomial To express a polynomial as a product of other polynomials.

Greatest monomial factor The GCF of the terms of a polynomial.

Special factoring patterns

Perfect square trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $\quad a^2 - 2ab + b^2 = (a - b)^2$
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$
Sum of cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 1 Factor: **a.** $3x^4 - 6x^3 + 12x^2$ **b.** $8a^3b - 12a^2b^2$

Solution **a.** $3x^4 - 6x^3 + 12x^2 = 3x^2(x^2 - 2x + 4)$ ← The GCF of the terms is $3x^2$.

b. $8a^3b - 12a^2b^2 = 4a^2b(2a - 3b)$ ← The GCF of the terms is $4a^2b$.

Factor each polynomial.

1. $y^2 + y$ **2.** $4x - 28$ **3.** $8a^4 - 14a^2b$

4. $6x^2 - 8x^3 - 10x^4$ **5.** $10x^8 + 15x^7 - 35x^5$ **6.** $11a^3b - 22a^2b^2 + 55ab^3$

Example 2 Factor: **a.** $z^2 + 8z + 16$ **b.** $9x^2 - 6xy + y^2$ **c.** $36m^2 - 49n^2$

Solution **a.** $z^2 + 8z + 16 = z^2 + 2(z)(4) + (4)^2$ ← perfect square trinomial

$= (z + 4)^2$

b. $9x^2 - 6xy + y^2 = (3x)^2 - 2(3x)(y) + y^2$ ← perfect square trinomial

$= (3x - y)^2$

c. $36m^2 - 49n^2 = (6m)^2 - (7n)^2$ ← difference of squares

$= (6m + 7n)(6m - 7n)$

Example 3 Factor $2a^5 - 162a$.

Solution Always begin by looking for the GCF of the terms. If the GCF is not 1, then factor the GCF out. The GCF of $2a^5$ and $-162a$ is $2a$.

$2a^5 - 162a = 2a(a^4 - 81)$

$= 2a[(a^2)^2 - (9)^2]$ ← difference of squares

$= 2a(a^2 + 9)(a^2 - 9)$

$= 2a(a^2 + 9)(a^2 - 3^2)$ ← difference of squares

$= 2a(a^2 + 9)(a + 3)(a - 3)$

NAME _____ DATE _____

4–5 Factoring Polynomials *(continued)*

Factor each polynomial.

7. $x^2 + 10x + 25$ **8.** $a^2 - 16a + 64$ **9.** $4y^2 - 12y + 9$

10. $4b^2 + 28b + 49$ **11.** $x^2 - 16$ **12.** $y^2 - 100$

13. $4k^2 - 25$ **14.** $9m^2 - 64$ **15.** $25x^2 + 20xy + 4y^2$

16. $81p^2 - 49q^2$ **17.** $3x^2 + 12x + 12$ **18.** $5c^3 + 30c^2d + 45cd^2$

19. $rt^2 - r$ **20.** $4x^2y - 36y$ **21.** $16n^4 - 1$

Example 4 Factor: **a.** $a^3 - 8$ **b.** $27x^3 + 1$

Solution **a.** $a^3 - 8 = a^3 - 2^3 = (a - 2)(a^2 + 2a + 4)$ ← difference of cubes

b. $27x^3 + 1 = (3x)^3 + 1^3 = (3x + 1)(9x^2 - 3x + 1)$ ← sum of cubes

Factor each polynomial.

22. $x^3 + 1$ **23.** $64 - a^3$ **24.** $t^3 + 125$ **25.** $1000c^3 - 27$

Example 5 Factor: **a.** $2a^3 - 3a^2 - 4a + 6$ **b.** $12x^3 + 4x^2y - 3x - y$

Solution **a.** The first and second terms have a common factor of a^2, and the third and fourth terms have a common factor of -2. Factor by grouping terms.

$$\begin{aligned} 2a^3 - 3a^2 - 4a + 6 &= (2a^3 - 3a^2) + (-4a + 6) \\ &= a^2(2a - 3) - 2(2a - 3) \quad \text{Common factor is } 2a - 3. \\ &= (a^2 - 2)(2a - 3) \quad \text{Factor out } (2a - 3). \end{aligned}$$

b. The first and third terms have a common factor of $3x$, and the second and fourth terms have a common factor of y. Factor by grouping terms.

$$\begin{aligned} 12x^3 + 4x^2y - 3x - y &= (12x^3 - 3x) + (4x^2y - y) \\ &= 3x(4x^2 - 1) + y(4x^2 - 1) \\ &= (3x + y)(4x^2 - 1) \leftarrow \text{difference of squares} \\ &= (3x + y)(2x + 1)(2x - 1) \end{aligned}$$

Factor each polynomial.

26. $a(b + 2) - 3(b + 2)$ **27.** $m(n - 2) - (2 - n)$ **28.** $20a^3 - 5a^2 + 8a - 2$

29. $10y^3 + 10y^2 + 3y + 3$ **30.** $9a^2b - 8a^2 - 9b + 8$ **31.** $5x^2y - 7x^2 - 7 + 5y$

Mixed Review Exercises

Write as a simplified polynomial.

1. $(a + 2)^2$ **2.** $(3a - 2)(4a + 3)$ **3.** $r^2s^2(4r - 5s)$

4. $(5a - 2) - (3 - a)$ **5.** $(x^2 - 2)(x + 5)$ **6.** $(-a)^2(2a^2)^3$